Lesson 1.1 • Recursively Defined Sequences

1. Find the common difference, \( d \), for each arithmetic sequence and the common ratio, \( r \), for each geometric sequence.
   a. 1.5, 1.0, 0.5, 0, −0.5, . . .  
   b. 0.0625, 0.125, 0.25, . . .  
   c. −1, 0.2, −0.04, 0.008, . . .

2. Write the first six terms of each sequence and identify each sequence as arithmetic or geometric.
   a. \( u_1 = -18 \)
      \( u_n = u_{n-1} + 6 \) where \( n \geq 2 \)
   b. \( u_1 = 0.5 \)
      \( u_n = 3u_{n-1} \) where \( n \geq 2 \)

3. Write a recursive formula to generate each sequence. Then find the indicated term.
   a. 17.25, 14.94, 12.63, 10.32, . . . Find the 15th term.
   b. −2, 4, −8, 16, . . . Find the 15th term.

4. Indicate whether each situation could be represented by an arithmetic sequence or a geometric sequence. Give the value of the common difference, \( d \), for each arithmetic sequence and of the common ratio, \( r \), for each geometric sequence.
   a. Phil rented an apartment for $850 a month. Each time he renewed his annual lease over the next 3 years, his landlord raised the rent by $50.

   b. Leora was hired as a first-year teacher at an annual salary of $30,000. She received an annual salary increase of 5% for each of the next 4 years.

5. Write a recursive formula for the sequence graphed at right. Find the 42nd term.
Lesson 1.2 • Modeling Growth and Decay

1. Find the common ratio for each sequence and identify the sequence as growth or decay. Give the percent increase or decrease for each.
   a. 42, 126, 378, 1134, . . .
   b. 19.2, 3.84, 0.768, 0.1536, . . .
   c. 90, 99, 108.9, 119.79, . . .
   d. 1800, 1080, 648, 388.8, . . .

2. Write a recursive formula for each sequence in Exercise 1. Use $u_0$ for the first term given and find $u_5$.

3. Factor each expression so that the variable appears only once. For example, $x + 0.05x$ factors into $x(1 + 0.05)$.
   a. $y - 0.19y$
   b. $2A - 0.33A$
   c. $u_{n-1} - 0.72u_{n-1}$
   d. $3u_{n-1} - 0.5u_{n-1}$

4. Write a recursive formula for the sequence 3, 8.5, 26, 77.5, . . .

5. Match each recursive formula to a graph.
   a. $u_0 = 35$
      $u_n = (1 - 0.3) \cdot u_{n-1}$ where $n \geq 1$
   b. $u_0 = 35$
      $u_n = (1 - 0.5) \cdot u_{n-1}$ where $n \geq 1$
   c. $u_0 = 35$
      $u_n = -0.5 + u_{n-1}$ where $n \geq 1$
   A. $u_n$
   B. $u_n$
   C. $u_n$
Lesson 1.3 • A First Look at Limits

1. For each sequence, find the value of \( u_1, u_2, \) and \( u_3 \). Identify the type of sequence (arithmetic, geometric, or shifted geometric) and tell whether it is increasing or decreasing.

   a. \( u_0 = 25 \)
      \[ u_n = u_{n-1} + 8 \] where \( n \geq 1 \)

   b. \( u_0 = 10 \)
      \[ u_n = 0.1u_{n-1} \] where \( n \geq 1 \)

   c. \( u_0 = 48 \)
      \[ u_n = u_{n-1} - 6.9 \] where \( n \geq 1 \)

   d. \( u_0 = 500 \)
      \[ u_n = (1 - 0.80)u_{n-1} + 25 \] where \( n \geq 1 \)

2. Solve.

   a. \( r = 0.9r + 30 \)
   b. \( s = 25 + 0.75s \)
   c. \( t = 0.82t \)

   d. \( v = 45 + v \)
   e. \( w = 0.60w - 20 \)
   f. \( z = 0.125z + 49 \)

3. Find the long-run value for each sequence.

   a. \( u_0 = 48 \)
      \[ u_n = 0.75u_{n-1} + 25 \] where \( n \geq 1 \)

   b. \( u_0 = 12 \)
      \[ u_n = 0.9u_{n-1} + 2 \] where \( n \geq 1 \)

   c. \( u_0 = 62 \)
      \[ u_n = (1 - 0.2)u_{n-1} \] where \( n \geq 1 \)

   d. \( u_0 = 45 \)
      \[ u_n = (1 - 0.05)u_{n-1} + 5 \] where \( n \geq 1 \)

4. Write a recursive formula for each sequence. Use \( u_0 \) for the first term given.

   a. 0, 20, 36, 48.8, . . .
   b. 100, 160, 226, 298.6, . . .

   c. 50, 36, 27.6, 22.56, . . .
   d. 40, 44, 50.4, 60.64, . . .
Lesson 1.4 • Graphing Sequences

1. Write five ordered pairs that represent points on the graph of each sequence.
   a. \( b_0 = 2 \)
      \( b_n = b_{n-1} + 8 \) where \( n \geq 1 \)
   b. \( b_0 = 10 \)
      \( b_n = 0.1b_{n-1} \) where \( n \geq 1 \)
   c. \( b_0 = 0 \)
      \( b_n = 2.5b_{n-1} + 10 \) where \( n \geq 1 \)
   d. \( b_0 = 150 \)
      \( b_n = 0.8b_{n-1} - 10 \) where \( n \geq 1 \)

2. Match each formula with a graph and identify the sequence as arithmetic or geometric.
   a. \( u_0 = 10 \)
      \( u_n = 1.5u_{n-1} \) where \( n \geq 1 \)
   b. \( u_0 = 30 \)
      \( u_n = u_{n-1} + 5 \) where \( n \geq 1 \)
   c. \( u_0 = 80 \)
      \( u_n = 0.75u_{n-1} \) where \( n \geq 1 \)
   A. \[ \begin{array}{c}
   \text{Graph A} \\
   \end{array} \]
   B. \[ \begin{array}{c}
   \text{Graph B} \\
   \end{array} \]
   C. \[ \begin{array}{c}
   \text{Graph C} \\
   \end{array} \]

3. Imagine the graphs of the sequences generated by these recursive formulas. Describe each graph using exactly three of these terms: arithmetic, geometric, shifted geometric, linear, nonlinear, increasing, decreasing.
   a. \( t_0 = 50 \)
      \( t_n = t_{n-1} - 10 \) where \( n \geq 1 \)
   b. \( a_0 = 1000 \)
      \( a_n = 0.7a_{n-1} + 100 \) where \( n \geq 1 \)
   c. \( u_0 = 35 \)
      \( u_n = u_{n-1} \cdot 1.75 \) where \( n \geq 1 \)
   d. \( t_0 = 150 \)
      \( t_n = (1 - 0.15)t_{n-1} \) where \( n \geq 1 \)
Lesson 1.5 • Loans and Investments

1. Assume that each of the sequences below represents a financial situation. Indicate whether each represents a loan or an investment, and give the principal and the deposit or payment amount.
   a. \( a_0 = 1000 \)
      \( a_n = (1 + 0.04)a_{n-1} + 100 \) where \( n \geq 1 \)
   b. \( a_0 = 130,000 \)
      \( a_n = \left(1 + \frac{0.0625}{4}\right)a_{n-1} - 1055 \) where \( n \geq 1 \)
   c. \( a_0 = 1825 \)
      \( a_n = \left(1 + \frac{0.075}{12}\right)a_{n-1} + 120 \) where \( n \geq 11 \)

2. For each financial situation represented by a sequence in Exercise 1, give the annual interest rate and the frequency with which interest is compounded.

3. Find the first month’s interest on each loan.
   a. $20,000 loan; 6% annual interest rate
   b. $1,650 loan; 4.6% annual interest rate

4. Write a recursive formula for each financial situation.
   a. You take out a home mortgage for $144,500 at 6.2%, compounded monthly, and make monthly payments of $990.

   b. You enroll in an investment plan through your job that deducts $225 from your monthly paycheck and deposits it into an account with an annual interest rate of 3.75%, compounded monthly.
LESSON 0.1 · Pictures, Graphs, and Diagrams

1. a. 2   b. \(\frac{3}{2}\)  c. \(-\frac{2}{3}\)

2. a. \(a = 12\)  b. \(b = 36\)  c. \(c = 33\)
   d. \(d = 78\)  e. \(w = 9\)  f. \(x = 280\)

3. a. \(\frac{7}{3}\)  b. \(-\frac{5}{2}\)  c. 1
   d. \(-1\)  e. \(-2\)  f. \(\frac{4}{5}\)

4. a. \(\frac{7}{12}\)  b. \(\frac{13}{10}\)  c. \(\frac{3}{5}\)
   d. \(\frac{5}{12}\)  e. \(\frac{7}{4}\)  f. \(\frac{2}{11}\)

LESSON 0.2 · Symbolic Representation

1. a. Subtract 7 from both sides.
   b. Divide both sides by 8.
   c. Multiply both sides by \(-11\).

2. a. \(a = 27\)  b. \(b = 9\)  c. \(c = 15\)
   d. \(d = -4\)  e. \(p = -\frac{7}{12}\), or \(-0.583\)
   f. \(q = \frac{3}{4}\), or 0.75

3. a. \(-21 + 3y\), or \(3y - 21\)
   b. \(-144q + 12q^2\), or \(12q^2 - 144q\)
   c. \(-7y^3 + 21y\)
   d. \(6r^2 - 15r\)
   e. \(48s^2 - 30s\)
   f. \(16x^3 - 120z\)

4. a. \(-25\)  b. 36  c. 26.4  d. 34

LESSON 0.3 · Organizing Information

1. a. \(2.6w - 10.4\)
   b. \(-4.6 - 2y\), or \(-2y - 4.6\)
   c. \(6.8s - 2.8t + 8.4\)
   d. \(3u - 26\)
   e. \(-8z + 12\), or \(12 - 8z\)

2. a. \(p = 3.5\), or \(p = \frac{7}{2}\)
   b. \(q = -13.3\), or \(-\frac{133}{10}\)
   c. \(s = -8\)
   d. \(z = -13\)

3. a. \(m^{12}\)
   b. \(p^{-10}\), or \(\frac{1}{p^{10}}\)
   c. \(81a^8b^4\)
   d. \(-10x\)

4. a. \(x^2 + 9x + 20\)
   b. \(4m^2 - 12m + 9\)
   c. \(49p^3 - 81\)

5. a. \(k = 0.25\); \(y = 0.25x\)
   b. 30 problems

LESSON 1.1 · Recursively Defined Sequences

1. a. \(d = -0.5\)  b. \(r = 2\)
   c. \(r = -0.2\)

2. a. \(-18, -12, -6, 0, 6, 12\); arithmetic
   b. 0.5, 1.5, 4.5, 13.5, 40.5, 121.5; geometric

3. a. \(u_1 = 17.25\) and \(u_n = u_{n-1} - 2.31\) where \(n \geq 2\);
   \(u_{15} = -15.09\)
   b. \(u_1 = -2\) and \(u_n = -2u_{n-1}\) where \(n \geq 2\);
   \(u_{15} = -32,768\)

4. a. Arithmetic; \(d = 50\)
   b. Geometric; \(r = 1.05\)

5. \(u_1 = 12\) and \(u_n = u_{n-1} - 4\) where \(n \geq 2\);
   \(u_{42} = -152\)

LESSON 1.2 · Modeling Growth and Decay

1. a. 3; growth; 200% increase
   b. 0.2; decay; 80% decrease
   c. 1.1; growth; 10% increase
   d. 0.6; decay; 40% decrease

2. a. \(u_0 = 42\) and \(u_n = 3u_{n-1}\) where \(n \geq 1\);
   \(u_5 = 10,206\)
   b. \(u_0 = 19.2\) and \(u_n = 0.2u_{n-1}\) where \(n \geq 1\);
   \(u_5 = 0.06144\)
   c. \(u_0 = 90\) and \(u_n = 1.1u_{n-1}\) where \(n \geq 1\);
   \(u_5 = 144.9459\)
   d. \(u_0 = 1800\) and \(u_n = 0.6u_{n-1}\) where \(n \geq 1\);
   \(u_5 = 139.968\)

3. a. \((1 - 0.19)y\), or 0.81y
   b. \((2 - 0.33)A\), or 1.67A
   c. \((1 - 0.72)u_{n-1}\), or 0.28\(u_{n-1}\)
   d. \((3 - 0.5)u_{n-1}\), or 2.5\(u_{n-1}\)

4. a. \(u_0 = 3\) and \(u_n = -3u_{n-1} + 0.5\) where \(n \geq 1\), or
   \(u_1 = 3\) and \(u_n = -3u_{n-1} + 0.5\) where \(n \geq 2\)

5. a. C  b. A  c. B

LESSON 1.3 · A First Look at Limits

1. a. \(u_1 = 33\), \(u_2 = 41\), \(u_3 = 49\); arithmetic; increasing
   b. \(u_1 = 1\), \(u_2 = 0.1\), \(u_3 = 0.01\); geometric; decreasing
   c. \(u_1 = 41.1\), \(u_2 = 34.2\), \(u_3 = 27.3\); arithmetic; decreasing
   d. \(u_1 = 125\), \(u_2 = 50\), \(u_3 = 35\); shifted geometric; decreasing
LESSON 2.1  Box Plots

1. a. Mean: 6.3; median: 7; mode: 7
   b. Mean: 182; median: 180; mode: none
   c. Mean: 27.7; median: 32; mode: none
   d. Mean: 8; median: 8.8; modes: 5.3, 9.2

2. a. 500  b. 250  c. 500
   d. 250  e. 750  f. 750

3. a. 2, 3, 6, 9, 10  b. 0, 30, 50, 80, 95
   c. 1, 2.5, 4, 8, 9  d. 16, 36, 52, 60, 70
   e. 16.7, 18.65, 20.95, 29.5, 33.9

4. a. Median = 18; range = 11; IQR = 8
   b. Median = 449.5; range = 766; IQR = 568

LESSON 1.4  Graphing Sequences

1. Sample answers:
   a. (0, 2), (1, 10), (2, 18), (3, 26), (4, 34)
   b. (0, 10), (1, 1), (2, 0.1), (3, 0.01), (4, 0.001)
   c. (0, 0), (1, 10), (2, 35), (3, 97.5), (253.75)
   d. (0, 150), (1, 110), (2, 78), (3, 52.4), (4, 31.92)

2. a. C; geometric  b. A; arithmetic  c. B; geometric

3. a. Arithmetic, linear, decreasing
   b. Shifted geometric, nonlinear, decreasing
   c. Geometric, nonlinear, increasing
   d. Geometric, nonlinear, decreasing

LESSON 1.5  Loans and Investments

1. a. Investment; principal: $1,000; deposit: $100
   b. Loan; principal: $130,000; deposit: $1,055
   c. Investment; principal: $1,825; deposit: $120

2. a. 4%; annually
   b. 6.25%, or $6.25\%$; quarterly
   c. 7.5%, or $\frac{15}{2}\%$; monthly

3. a. $100.00  b. $6.33

4. a. $u_0 = 144,500$ and $u_n = (1 + \frac{0.0625}{12})u_{n-1} - 990$
   b. $u_0 = 0$ and $u_n = (1 + \frac{0.0375}{12})u_{n-1} + 225$
   where $n \geq 1$

LESSON 2.2  Measures of Spread

1. a. $\bar{x} = 18$; deviations: $\{-5.6, 8.3, -8.2, 15.9, -10.4\}$
   b. $\bar{x} = 421$; deviations: $\{-186, -8, 84, -310, 279, 205, -64\}$

2. a. $\bar{x} = 41$ in.; $s = 8.9$ in.
   b. $\bar{x} = 12.5$ cm; $s = 4.80$ cm
   c. $\bar{x} = 13.80$; $s = 2.88$

3. Sample answer: $\{16, 9, 13, 9, 13\}$

4. a. 105  b. None  c. 5, 95

LESSON 2.3  Histograms and Percentile Ranks

1. a. 25%
   b. Ages of Family Members

2. a. Bin width: 8; 29 values; bin 16–24
   b. Bin width: 12; 72 values; bin 24–36
   c. Bin width: 35; 1168 values; bin 140–175

3. a. 54th percentile  b. 62nd percentile

LESSON 3.1  Linear Equations and Arithmetic Sequences

1. a. $u_n = 18.25 - 4.75n$  b. $t_n = 100n$

2. a. $u_1 = 35$ and $u_n = u_{n-1} - 7$ where $n \geq 2$
   b. Slope: $-7$; $y$-intercept: 42
   c. $y = -7x + 42$

3. a. 3  b. $-1$  c. 0.6

4. a. $y = 11 + 9x$  b. $y = -7.5 - 12.5x$

LESSON 3.2  Revisiting Slope

1. a. 3  b. $\frac{7}{5}$  c. $\frac{5}{3}$

2. a. $-2.5$  b. $-4$  c. $-0.3$

3. a. $y = 14$  b. $a = 390$

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